

1: Vector Spaces

(1.19): Conditions for a vector space.

(1.23, 1.24): \mathbb{F}^S is a vector space.

(1.34): Conditions for a subspace.

(1.36, 1.44): Sums of subspaces, conditions for a sum to be a direct sum.

(1C - Exercise 7, 8, 10-13): When something is and isn't a subspace.

(1C - Exercise 20-23): Working with direct sums.

2: (Finite-Dimensional) Vector Spaces

(2.13): The vector space of polynomials.

(2.17-2.22): Linear independence/dependence, the Linear Dependence Lemma.

(2A - Exercise 6-11): Finding/showing linear independence/dependence.

(2.31, 2.33): Spanning sets can be reduced to a basis; linearly independent sets can be extended to a basis.

(2B - Exercise 1-4): Working with basis.

(2B - Exercise 6, 7): What a basis can and can't accomplish.

(2B - Exercise 8): Significance of direct sums on basis sets.

(2C - Exercise 2-8): More working with basis.

3: Linear Maps

(3.2): Conditions for a linear map.

(3.5): Linear maps are determined by what they do to the basis of the domain.

(3A - Exercise 1, 2): Working with linear maps.

(3A - Exercise 8-10): When things fail to be linear maps.

(3.13): Examples of null spaces.

(3.15): Definition of injectivity.

(3.16): Injectivity if and only if the null space is $\{0\}$.

(3.18): Examples of ranges of linear maps.

(3.20): Definition of surjectivity.

(3.22): Fundamental Theorem of Linear Maps.

(3B - Exercise 3-8): Working with linear maps in a more advanced context.

(3B - Exercise 11): Proof involving injectivity.

(3B - Exercise 13, 14, 26): Proofs involving surjectivity.

(3B - Exercise 16-25): Proofs involving linear maps and subspaces.

(3.32, 3.33, 3.34): Matrix representation of a linear map.

(3.41-3.50): Matrix multiplication.

(3C - Exercise 3-6): Significance of basis sets to matrix representations.

(3C - Exercise 12-15): Working with matrix arithmetic.

(3.53-3.56): Invertibility definition and conditions.

(3.58, 3.59): Isomorphism definition and conditions.

(3.62-3.63): Matrix representation of vectors.

(3.69, 3.70): Useful facts about operators and an application.

(3D - Exercise 1-6): Proofs involving invertibility.

(3D - Exercise 8-11): Proofs involving operators (maps from a vector space to itself).

(3D - Exercise 14, 15): Proofs involving isomorphism.

5: Eigenvalues, Eigenvectors, and Invariant Subspaces

(5.2): Invariant subspace definition.

(5.5, 5.6): Eigenvalue definition and conditions.

(5.7, 5.8): Eigenvector definition and examples.

(5.10, 5.13): Properties of eigenvectors and eigenvalues.

(5A - Exercise 1-6): Proofs showing something is invariant.

(5A - Exercise 7-12): Finding eigenvalues and eigenvectors.

(5A - Exercise 17, 18, 20, 22, 30): Basic exercises around eigenvalues and eigenvectors.

(5A - Exercise 14-16, 19, 21, 24-26, 31, 32): Advanced exercises around eigenvalues and eigenvectors.

(5.21): There is always an eigenvalue for operators on complex vector spaces.

(5.22, 5.23, 5.24, 5.25): Matrix representations of operators, nomenclature.

(5.25, 5.30, 5.32): Upper-triangular matrix definition and uses.

(5B - Exercise 2, 3): More eigenvalue problems.

(5B - Exercise 5, 6, 9-11): Consequences of "linear operator polynomials".