<u>Math 125 Preliminaries</u>

There are several topics that you are expected to know to be prepared for this course. These include:

- Common Errors
- Rules for Exponents
- Simplifying Complex Fractions
- Combining Rational Expressions

If you make a mistake on your exam on any of these topics, you will automatically lose half the points on the problem. The error will be marked by an asterisk * to indicate that one of these types of errors has been made.

Common Errors

Incorrect	Correct
$(a+b)^2 = a^2 + b^2$	$(ab)^2 = a^2b^2$
$\sqrt{a^2+b^2} = a+b$	$\sqrt{a^2b^2} = ab$ (for $a>0$ and $b>0$)
$\dfrac{\cancel{a}+b}{\cancel{a}+c} = \dfrac{b}{c}$	$\dfrac{\cancel{a}b}{\cancel{a}c} = \dfrac{b}{c}$

For example:

$(x+4)^2 \neq x^2 + 4^2$ but $(4x)^2 = 4^2x^2 = 16x^2$

$\sqrt{x^2+4^2} \neq x+4$ but $\sqrt{4^2x^2} = 4x$ (for $x>0$)

$\dfrac{5+\cancel{x}}{2+\cancel{x}} \neq \dfrac{5}{2}$ but $\dfrac{5x}{2x} = \dfrac{5}{2}$

As you see, operations for multiplication don't translate to addition.

For the square of a binomial, remember *there is a middle term*:

$$(a+b)^2 = a^2 + 2ab + b^2$$

The middle term is twice the product of the first and second term.
For example:

$$(x+4)^2 = x^2 + 2 \cdot 4x + 4^2 = x^2 + 8x + 16$$

Memorize the formula rather than foiling each time you see the square of a binomial.

The third example (canceling terms) is a common error, but one that is very important to avoid.

You can see in this example why the cancellation does not work:

$$\frac{3+2}{4+2} = \frac{5}{6} \text{ not } \frac{3}{4}.$$

Here is another example of a cancellation error:

$$\frac{4x + 2\cancel{(x+4)}}{5\cancel{(x+4)}}$$

This cancellation is correct:

$$\frac{4x\cancel{(x+4)}}{5\cancel{(x+4)}} = \frac{4x}{5}$$

What you should remember is that all terms must be multiplied for you to cancel.
Visually, if + (or -) appears by itself outside any parenthesis you cannot cancel.

Here is an example of how to proceed to cancel a common factor:

$$\frac{6+2\sqrt{5}}{4} = \frac{2(3+\sqrt{5})}{4} = \frac{3+\sqrt{5}}{2}$$

<u>Rules for Exponents</u>

Rules:	Examples:
$x^0 = 1 \quad (x \neq 0)$	$5^0 = 1$
$x^{-n} = \dfrac{1}{x^n}$	$2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

$$\frac{1}{x^{-n}} = x^n \qquad \frac{1}{3^{-3}} = 3^3 = 27$$

$$x^n x^m = x^{n+m} \qquad x^2 x^3 = x^5$$

$$\frac{x^n}{x^m} = x^{n-m} \qquad \frac{x^5}{x^{-3}} = x^{5-(-3)} = x^8$$

$$\left(x^n\right)^m = x^{nm} \qquad \left(x^2\right)^3 = x^6$$

$$(xy)^n = x^n y^n \qquad (2x)^4 = 2^4 x^4 = 16x^4$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n} \qquad \left(\frac{5}{x}\right)^2 = \frac{5^2}{x^2} = \frac{25}{x^2}$$

Example:

$$\frac{(2x^3y^{-2})^2}{x^{-3}y^4} = \frac{4x^6y^{-4}}{x^{-3}y^4} = 4x^{6-(-3)}y^{-4-4} = 4x^9y^{-8} = \frac{4x^9}{y^8}$$

Only positive exponents should appear in your final answer.

Complex Fractions

A complex fraction contains a fraction in the numerator and/or denominator.
When a complex fraction appears, you will be expected to simplify it into a simple fraction.

Example:

$$\frac{(\frac{2}{5})}{3} = \frac{2}{5}\cdot\frac{1}{3} = \frac{2}{15} \qquad \text{Or: } \frac{(\frac{2}{5})}{3} = \frac{(\frac{2}{5})}{(\frac{3}{1})} = \frac{2}{5}\cdot\frac{1}{3} = \frac{2}{15}$$

In general, $\frac{(\frac{a}{b})}{c} = \frac{a}{b}\cdot\frac{1}{c} = \frac{a}{bc}$. In other words, if you have a fraction over a denominator, you *multiply the denominators.*

Example:

$$\frac{1-\frac{\sqrt{3}}{2}}{5}$$

To simplify, we multiply each term by the least common denominator of all the fractions that appear in the numerator and denominator. We do this to eliminate the denominator(s). In this case there is only one denominator, so we multiply the numerator and denominator by 2, which in turn will multiply each of the terms by 2:

$$\frac{1-\frac{\sqrt{3}}{2}}{5}\cdot\frac{2}{2}=\frac{1\cdot 2-\frac{\sqrt{3}}{2}\cdot 2}{5\cdot 2}=\frac{2-\sqrt{3}}{10}$$

Example:

$$\frac{\frac{3}{x}-\frac{4}{x^2}}{\frac{5}{2x}}$$

The least common denominator is $2x^2$.

$$\frac{\frac{3}{x}-\frac{4}{x^2}}{\frac{5}{2x}}\cdot\frac{2x^2}{2x^2}=\frac{\frac{3}{x}\cdot 2x^2-\frac{4}{x^2}\cdot 2x^2}{\frac{5}{2x}\cdot 2x^2}=\frac{6x-8}{5x}.$$

Of course, we can't cancel the $x's$ in the final answer.

An alternate method is done by combining fractions that appear in the numerator and/or combining fractions that appear in the denominator. The method above is quicker, and after a little practice, easier.

Combining Rational Expressions

Rational expressions are fractions which contain variable(s) raised to positive integer powers.

Let's first start with combining fractions that contain just numbers.

To add or subtract fractions, we must first rewrite each fraction using the Least Common Denominator (LCD). We can then combine the numerators.

$$\frac{2}{5}+\frac{3}{4}=\frac{2}{5}\cdot\frac{4}{4}+\frac{3}{4}\cdot\frac{5}{5}=\frac{8}{20}+\frac{15}{20}=\frac{8+15}{20}=\frac{23}{20}$$

$$4+\frac{2}{3}=\frac{4}{1}\cdot\frac{3}{3}+\frac{2}{3}=\frac{12+2}{3}=\frac{14}{3}$$

The same procedure applies when we combine rational expressions.

Example:

$$\frac{3}{4(x+2)}-\frac{5}{2x}$$

The denominators contain the numbers 4 and 2, and the variable factors $x, x+2$.
To find the LCD, first find the least common multiple of the numbers and then multiply by the different variable factors. So in this case, the LCD is $4x(x+2)$. We now rewrite each fraction separately with the LCD. To obtain $4x(x+2)$ from $4(x+2)$, we must multiply by x. So we multiply the numerator and denominator of the first fraction by $\frac{x}{x}$.
For the second fraction, to obtain $4x(x+2)$ from $2x$, we must multiply by $2(x+2)$. So we multiply the second fraction by $\frac{2(x+2)}{2(x+2)}$.

$$\frac{3}{4(x+2)}\cdot\frac{x}{x}-\frac{5}{2x}\cdot\frac{2(x+2)}{2(x+2)}=\frac{3x}{4x(x+2)}-\frac{10(x+2)}{4x(x+2)}=\frac{3x-10(x+2)}{4x(x+2)}=$$

$$=\frac{3x-10x-20}{4x(x+2)}=\frac{-7x-20}{4x(x+2)}$$

Be careful with the subtraction in the numerator. A common mistake is to obtain $3x-10x+20$ rather than $3x-10x-20$.

Example:

$$\frac{7}{6(x-2)^2}-\frac{5}{4x(x-2)}$$

The denominators contain the numbers 6 and 4, and the variable factors $x, x-2, (x-2)^2$. If a variable factor appears more than once (in this case, $(x-2)$ and $(x-2)^2$), choose the highest power. So in this case, the LCD is $12x(x-2)^2$. For the first fraction, to obtain $12x(x-2)^2$ from $6(x-2)^2$, we must multiply by $2x$. So we multiply the first fraction by $\frac{2x}{2x}$. Similarly, we multiply the second fraction by $\frac{3(x-2)}{3(x-2)}$.

$$\frac{7}{6(x-2)^2}\cdot\frac{2x}{2x}-\frac{5}{4x(x-2)}\cdot\frac{3(x-2)}{3(x-2)}=\frac{14x-15(x-2)}{12x(x-2)^2}=\frac{14x-15x+30}{12x(x-2)^2}=$$

$$\frac{-x+30}{12x(x-2)^2}.$$

For the problems in this course, you should always leave the denominator in factored form.