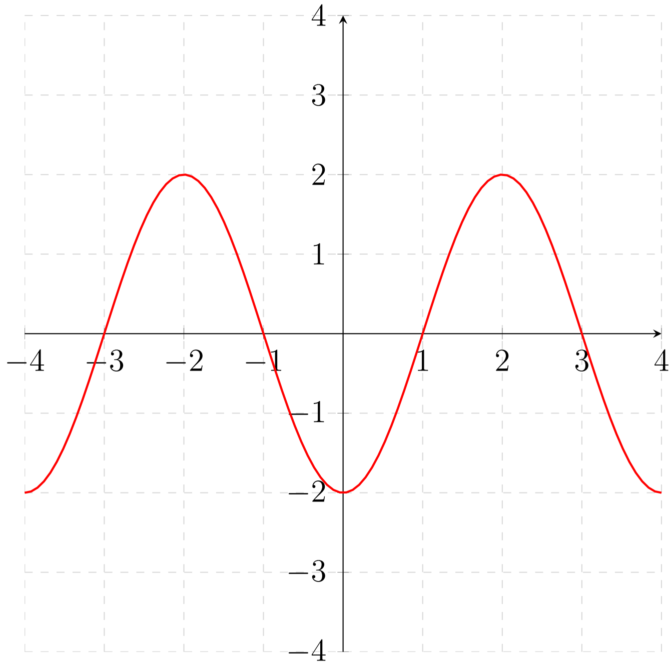
1. Graph the following function: $f(x) = 2\cos(\frac{\pi}{2}x - \pi)$



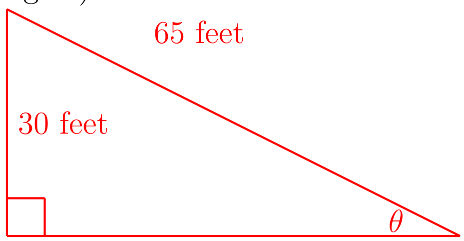
2. Find the exact value, if possible:

   (a) $\cos(\cos^{-1} 0.6)$
   $\cos^{-1} .6$ is some angle in the first quadrant because the range of $\cos^{-1}$ is $[0, \pi]$. Thus $\cos(\cos^{-1} .6) = .6$ since the range of cos is $[0, 2\pi]$.

   (b) $\sin^{-1}\left(\sin \frac{3\pi}{2}\right)$
   $\sin \frac{3\pi}{2} = -1$; $\sin^{-1}(-1) = -\frac{\pi}{2}$ since the range of $\sin^{-1}$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

   (c) $\cos(\tan^{-1} 1)$
   $\tan^{-1} 1 = \frac{\pi}{4}$ since the range of $\tan^{-1}$ is $(-\frac{\pi}{2}, \frac{\pi}{2})$. Finally, $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

3. A kite flies at a height of 30 feet when 65 feet of string is out. If the string is in a straight line, find the angle that it makes with the ground (round to the nearest degree).



To compute θ, refer to the picture. $\sin\theta = \dfrac{30}{65} \implies \theta = \sin^{-1}\dfrac{30}{65} = 27.4864 = 27°$.

4. Verify the identities:

(a) $\dfrac{1+\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$

$$\frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$$

(b) $\dfrac{\cos\theta}{1+\sin\theta} + \dfrac{1+\sin\theta}{\cos\theta} = 2\sec\theta$

$$\begin{aligned}
\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} &= \left(\frac{1-\sin\theta}{1-\sin\theta}\right)\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} \\
&= \frac{(1-\sin\theta)\cos\theta}{1-\sin^2\theta} + \frac{1+\sin\theta}{\cos\theta} \\
&= \frac{(1-\sin\theta)\cancel{\cos\theta}}{\cos^{\cancel{2}}\theta} + \frac{1+\sin\theta}{\cos\theta} \\
&= \frac{1-\sin\theta}{\cos\theta} + \frac{1+\sin\theta}{\cos\theta} \\
&= \frac{1-\sin\theta+1+\sin\theta}{\cos\theta} \\
&= \frac{2}{\cos\theta} \\
&= 2\sec\theta
\end{aligned}$$

5. Find the exact value, using either the sum, difference, or half-angle identities:

   (a) $\cos 15°$

   $\cos 15° = \cos(45° - 30°) = \cos 45 \cos 30 + \sin 45 \sin 30 = \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}$
   $= \frac{\sqrt{6}+\sqrt{2}}{4}$

   (b) $\cos 75°$

   $\cos 75° = \cos(45° + 30°) = \cos 45 \cos 30 - \sin 45 \sin 30 = \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}$
   $= \frac{\sqrt{6}-\sqrt{2}}{4}$

   (c) $\cos 112.5°$

   First, identify that $112.5°$ is in the second quadrant. Hence the cosine will be a negative quantity. Now, apply the half angle formula:

   $$\cos\frac{225}{2} = -\sqrt{\frac{1+\cos 225}{2}} = -\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$$
   $$= -\sqrt{\frac{2+\sqrt{2}}{4}} = -\frac{\sqrt{2+\sqrt{2}}}{2}$$

6. Write an equivalent expression for $\sin^4 x$ that does not contain powers of trigonometric functions greater than 1.

   $\sin^2 x = \frac{1-\cos 2x}{2}$; apply this repeatedly.

   $\sin^4 x = \sin^2 x \sin^2 x = \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right) = \frac{1}{4}\left(1 - 2\cos 2x + \cos^2 2x\right)$

   Now we must also reduce $\cos^2 2x$; so apply the formula $\cos^2 x = \frac{1+\cos 2x}{2}$ to get $\cos^2 2x = \frac{1+\cos(2\cdot 2x)}{2}$; the solution is thus $\frac{1}{4}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)$

7. Solve for all x:

(a) $2\cos^2 x + 3\sin x = 0$

Know that $\cos^2 x = 1 - \sin^2 x$: hence

$$\begin{aligned} 2(1-\sin^2 x) + 3\sin x &= 0 \\ 2 - 2\sin^2 x + 3\sin x &= 0 \\ -2\sin^2 x + 3\sin x + 2 &= 0 \\ 2\sin^2 x - 3\sin x - 2 &= 0 \\ (2\sin x + 1)(\sin x - 2) &= 0 \end{aligned}$$

The solutions to each factor separately:
$2\sin x + 1 = 0$ when $\sin x = \frac{-1}{2}$; $x = -\frac{\pi}{6} + 2\pi k, \frac{7\pi}{6} + 2\pi k$ for all $k \in \mathbb{Z}$.
$\sin x - 2 = 0$ can never happen.

(b) $\cos 2x + 3\sin x - 2 = 0$

Know that $\cos 2x = 1 - 2\sin^2 x$: hence

$$\begin{aligned} 1 - 2\sin^2 x + 3\sin x - 2 &= 0 \\ -2\sin^2 x + 3\sin x - 1 &= 0 \\ 2\sin^2 x - 3\sin x + 1 &= 0 \\ (2\sin x - 1)(\sin x - 1) &= 0 \end{aligned}$$

The solutions to each factor separately:
$2\sin x - 1 = 0$ when $\sin x = \frac{1}{2}$; $x = \frac{\pi}{6} + 2\pi k, \frac{5\pi}{6} + 2\pi k$ for all $k \in \mathbb{Z}$.
$\sin x - 1 = 0$ when $\sin x = 1$; $x = \frac{\pi}{2} + 2\pi k$ for all $k \in \mathbb{Z}$.

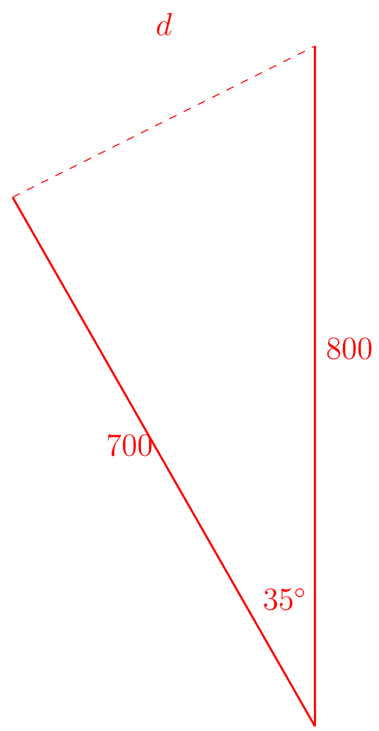
8. Solve triangle ABC if $A = 40°$, $a = 54$, and $b = 62$. Round lengths to the nearest tenth and angles to the nearest degree.

Apply the law of sines: $\dfrac{54}{\sin 40°} = \dfrac{62}{\sin B}$; so $\sin B = \dfrac{62}{54}\sin 40°$
Hence $\sin B = .738015$. There are two angles B from 0 to π such that $\sin B = .738015$; the first is $47.5626°$, and the second is its supplement $180° - 47.5626° = 132.437°$. We will work separately:

$B = 47.5626°$ case: so $C = 92.4374°$, and thus $\dfrac{c}{\sin 92.4374°} = \dfrac{a}{\sin 40°}$ means that $c = \dfrac{a}{\sin 40°}\sin 92.4374° = 83.9331°$.

$B = 132.437°$ case: so $C = 7.563°$, and thus $\dfrac{c}{\sin 7.563°} = \dfrac{a}{\sin 40°}$ means that $c = \dfrac{a}{\sin 40°}\sin 7.563° = 11.057$.

9. Two airplanes leave an airport at the same time on different runways. One flies directly north at 400 miles per hour. The other flies north-northwest at a 35° to the first airplane at 350 miles per hour. How far apart are the two planes after two hours?



Consult the diagram. We will apply the law of cosines: $d^2 = 800^2 + 700^2 - 2(800)(700)\cos 35°$;

$$d^2 = 1130000 - 1120000\cos 35° = 212550$$
$$d = \sqrt{212550} = 461.031$$

Trigonometric Identities

Sum Identities

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

Difference Identities

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Double-Angle Identities

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Power-Reducing Identities

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$
$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

Half-Angle Identities

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$
$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$
$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1 + \cos\theta} = \frac{1 - \cos\theta}{\sin\theta}$$