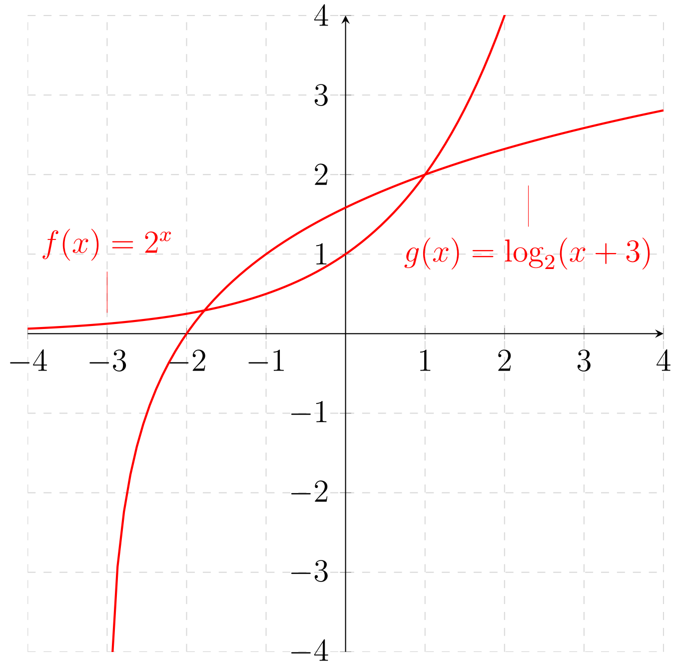
1. State the domain and range of the following functions:

   (a) $f(x) = 2^x$
   The domain is $\mathbb{R}$ and the range is $(0, \infty)$

   (b) $g(x) = \log_2(x + 3)$
   The domain is $(-3, \infty)$ and the range is $\mathbb{R}$

   Graph both $f(x)$ and $g(x)$ below:



2. Solve the following equations for x:

   (a) $\log_2(x) + \log_2(x - 7) = 3$
   Group together:

$$\log_2(x) + \log_2(x - 7) = 3 \implies \log_2(x(x - 7)) = 3$$

   Raise both sides by base 2:

$$x(x - 7) = 2^3 \implies x^2 - 7x = 8$$

   Solving the quadratic $x^2 - 7x - 8 = 0$ yields $x = -1$ and $x = 8$. Checking the answers yields $x = 8$ as the only valid answer.

   (b) $\ln(x + 2) - \ln(4x + 3) = \ln(\frac{1}{x})$
   Group together:

$$\ln(\frac{x + 2}{4x + 3}) = \ln(\frac{1}{x})$$

   This amounts to the equation $\dfrac{x + 2}{4x + 3} = \dfrac{1}{x}$; which in turn gives us $x(x+2) = 4x+3$. Working this out: $x^2 + 2x = 4x + 3 \implies x^2 - 2x - 3 = 0$ which has solutions $x = -1$ and $x = 3$. Checking the answers yields $x = 3$ as the only valid answer.

3. Carbon-14 has a half life of 5730 years. In 1947, earthware jars containing what are known as the *Dead Sea Scrolls* were found by an Arab Bedouin herdsman. Analysis indicated that the scroll wrappings contained 77% of their original Carbon-14.

   (a) Find the exponential decay model for Carbon-14 using the half life of 5730 years.
   If the half life is 5730 this means that the following equation is satisfied by the amount of Carbon-14 as a function of time:

   $$.5 = 1e^{5730 \cdot k}$$

   Solving for k yields $k = \dfrac{\ln\left(\frac{1}{2}\right)}{5730} = -0.000120968$ so our model is

   $$A(t) = A_0 e^{-0.000120968 \cdot t}$$

   (b) Using your model, estimate the age of the scrolls.
   Using the model we had from earlier we have that $.77 = 1e^{kt}$. In other words $\ln(.77) = kt$ or $t = \dfrac{\ln(.77)}{k}$. Using our previously computed values for k we have $t = 2160.61$: this implies that in 1947 the scrolls were 2160.61 years old.

4. The following two equations model the population (in millions) for Canada and Uganda t years after 2010:

   $$\text{Canada: } P = 33.1e^{0.009t}$$
   $$\text{Uganda: } P = 28.2e^{0.034t}$$

   When will Uganda's population exceed Canada's population?
   First we will compute when Uganda's population will match Canada's population. Since Uganda has a higher growth rate, this will also be the year when Uganda's population first exceeds Canada's population. So, our question amounts to solving the following equation for t:

   $$\begin{aligned} 33.1e^{0.009t} &= 28.2e^{0.034t} \\ \frac{33.1}{28.2} &= \frac{e^{0.034t}}{e^{0.009t}} \\ &= e^{0.025t} \\ \ln\left(\frac{33.1}{28.2}\right) &= 0.025t \\ \frac{\ln\left(\frac{33.1}{28.2}\right)}{0.025} &= t = 6.41 \end{aligned}$$

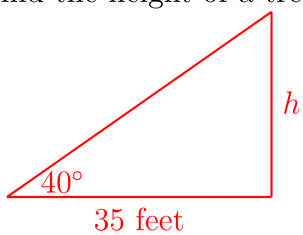
   This means by 2017 Uganda will have exceeded Canada's population.

5. Assuming the Earth is a perfect sphere of radius 3,959 miles (6,371 kilometers), what is the linear speed of someone sitting on the equator as the Earth spins? (Use whatever units you please). The angular speed of someone on the Earth's equator is $\frac{2\pi \text{ radians}}{\text{day}}$. Working the unit conversions:

$$\frac{2\pi \text{ radians}}{\text{day}} \cdot \frac{1 \text{ revolution}}{2\pi \text{ radians}} \cdot \frac{2\pi \cdot 3959 \text{ miles}}{1 \text{ revolution}} = \frac{7918\pi \text{ miles}}{\text{day}} = \frac{24875.1 \text{ miles}}{\text{day}}$$

$$\frac{2\pi \text{ radians}}{\text{day}} \cdot \frac{1 \text{ revolution}}{2\pi \text{ radians}} \cdot \frac{2\pi \cdot 6371 \text{ kilometers}}{1 \text{ revolution}} = \frac{12742\pi \text{ kilometers}}{\text{day}} = \frac{40030.2 \text{ kilometers}}{\text{day}}$$

6. At a certain time of day, the angle of elevation of the sun is 40°. To the nearest foot, find the height of a tree whose shadow is 35 feet long at that moment.



$$\tan(40°) = \frac{h}{35} \implies h = 35 \cdot \tan(40°) = 24.51$$

To the nearest foot, the tree is 25 feet tall.

7. Find the **exact** value of the following:

(a) $\sin(\frac{-\pi}{3})$

$\frac{-\sqrt{3}}{2}$

(b) $\cos(\frac{19\pi}{3})$

$\frac{1}{2}$

(c) $\tan(\frac{-8\pi}{3})$

$\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$

(d) $\sec(\frac{11\pi}{4})$

$\frac{1}{\frac{-\sqrt{2}}{2}} = -\sqrt{2}$

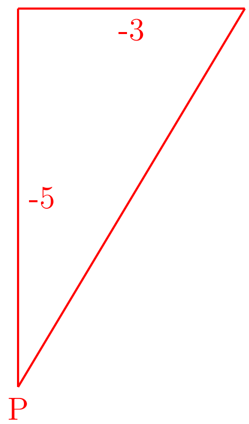
(e) $\csc(\frac{5\pi}{3})$

$\frac{1}{\frac{-\sqrt{3}}{2}} = \frac{-2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

(f) $\cot(\frac{5\pi}{6})$

$\frac{\frac{-\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

8. Let $P = (-3, -5)$ be a point on the terminal side of θ. What is the value of $\sin(\theta)$ and $\cos(\theta)$?



By the Pythagorean theorem, the hypoteneuse must be $\sqrt{34}$ in length. Thus $\sin(\theta) = \frac{-5}{\sqrt{34}} = \frac{-5\sqrt{34}}{34}$, and $\cos(\theta) = \frac{-3}{\sqrt{34}} = \frac{-3\sqrt{34}}{34}$.

9. Find the exact value of $\sin(\theta)$ and $\cos(\theta)$ provided $\tan(\theta) = \frac{-1}{3}$ and $\sin(\theta) > 0$.

Tangent is the ratio of the opposite side to the adjacent side. Thus the opposite side is length 1, with either positive or negative sign; similarly, the adjacent side is length 3 with either positive or negative sign. Thus the hypoteneuse must be length $\sqrt{10}$. The two possibilities here are that $\sin(\theta) = \frac{-1}{\sqrt{10}}, \cos(\theta) = \frac{3}{\sqrt{10}}$ or $\sin(\theta) = \frac{1}{\sqrt{10}}, \cos(\theta) = \frac{-3}{\sqrt{10}}$. However, we are given that $\sin(\theta) > 0$; so only the second possibility can be true. Thus $\sin(\theta) = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$ and $\cos(\theta) = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10}$

10. Find the exact value of $\cot(\theta)$ provided $\sin(\theta) = \frac{5}{13}$ and θ is in quadrant II.

Knowing $\sin(\theta) = \frac{5}{13}$ tells us that the opposite side of the triangle is $+5$ and the hypoteneuse is $+13$ (thus, by the Pythagorean theorem the adjacent side is ± 12). Thus our triangle can have an adjacent side (or cosine) of either $+12$ or -12: but knowing θ is in quadrant II it must be the case that the cosine is $\frac{-12}{13}$. Thus $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{\frac{-12}{13}}{\frac{5}{13}} = \frac{-12}{5}$.