M125 Final Exam Review

1. Determine whether the following relation is a function. Give the domain and range for the relation. (section 1.2)
   $\{(2, 4), (1, 3), (-2, 3), (0, -2), (1, 5)\}$

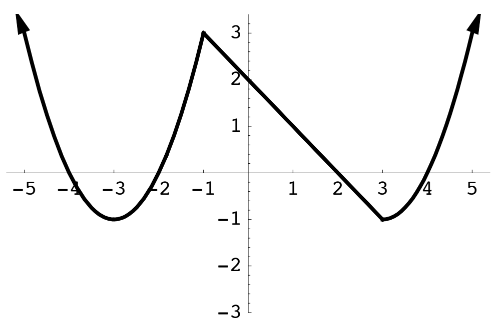
2. Determine whether the following equation defines y as a function of x. (section 1.2)
   $x^2 + y^2 = 25$

3. Graph the following piecewise-defined function: (section 1.3)
   $$f(x) = \begin{cases} x^2, & x < 1 \\ -x - 1, & x \geqslant 1 \end{cases}$$

4. Find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$ for the following functions: (section 1.3)
   a) $f(x) = -x^2 - 2x + 3$.
   b) $f(x) = \frac{1}{5x}$.

5. Determine whether $f(x) = 2x^4 - x^3 + 1$ is even, odd, or neither. Justify your answer. (section 1.3)

6. Given the graph of f below, determine each of the following. (section 1.3)



   a) the domain of f. b) the range of f. c) the x-intercepts (if any). d) the y-intercept (if any).
   e) the intervals on which f is increasing (if any), decreasing (if any), or constant (if any).
   f) the numbers, if any, at which f has a relative maximum. What are these relative maxima?
   g) the numbers, if any, at which f has a relative minimum. What are these relative minima?

7. Write the equation in slope-intercept form of the line that is a) parallel and b) perpendicular to the line $-2x + 3y = 7$ and passes through the point (-2, 3). (section 1.5)

8. Find the average rate of change of the function $f(x) = x^2 - 2x$ from $x_1 = -2$ to $x_2 = 3$. (section 1.5)

9. Graph the function $y = -2|x+1| + 2$. (section 1.6)

10. Find the domains of the following functions: a) $f(x) = \frac{2x}{x^2-2x-8}$. b) $g(x) = \sqrt{4-2x}$. (section 1.7)

11. Find $(f \circ g)(x)$ and the domain of $(f \circ g)(x)$ where $f(x) = \frac{x}{x+5}$ and $g(x) = \frac{6}{x}$. (section 1.7)

12. Given $f(x) = \frac{2x}{3x-1}$ find $f^{-1}(x)$. (section 1.8)

13. Find the midpoint of the line segment connecting the points $(-2, -1)$ and $(-8, 6)$ and then find the distance between these two points. (section 1.9)

14. Find the center and radius of the following equation of a circle by putting it in standard form and then graph. (section 1.9)
$x^2 + 4x + y^2 - 8y + 16 = 0$

15. Write $\frac{2-4i}{4+3i}$ in standard $a + b\,i$ form. (section 2.1)

16. Solve the quadratic equation $3x^2 = 4x - 6$ using the quadratic formula. If the solution(s) are complex, write in standard $a + b\,i$ form. (section 2.1)

17. Graph the function $f(x) = x^2 + 4x + 1$. (section 2.2)

18. A rectangular garden is to be fenced off and divided into 3 equal regions by fencing parallel to one side of the garden. 200 feet of fencing is used. Find the dimensions of the garden that maximize the total area enclosed. What is the maximum area? (section 2.2)

19. Use the Leading Coefficient Test to determine the end behavior of the graph of
$f(x) = -12x^5 + 3x^3 - 2x^2 + 14$. (section 2.3)

20. Use the Intermediate Value Theorem to show that the polynomial function $f(x) = x^3 + x^2 - 2x + 1$ has a real zero between -3 and -2. (section 2.3)

21. Determine the x-intercepts, the y-intercept, and use these along with points between and beyond the x-intercepts to graph the polynomial function $f(x) = -2(x+2)(x-1)(x-2)$. (section 2.3)

22. Use the Rational Zero Theorem to list all possible rational zeros of the function
$f(x) = 3x^4 - 11x^3 - 3x^2 - 6x + 8$. (section 2.5)

23. Find all zeros of the polynomial function $f(x) = x^3 - 6x^2 + 10x - 3$. (section 2.5)

24. Find an nth-degree polynomial function with real coefficients satisfying the given conditions.
$n = 3$; 6 and $-5 + 2i$ are zeros; $f(2) = -636$. (section 2.5)

25. Graph the rational function $f(x) = \frac{x^2+x-12}{x^2-4}$. List the intercepts, the vertical asymptote(s) (if any), and the horizontal asymptote (if any). (section 2.6)

26. Find the slant asymptote of $f(x) = \frac{3x^3-8x^2+16x-6}{x^2-2x+4}$. (section 2.6)

27. Solve the polynomial inequality $3x^2 + 7x \geq 6$. (section 2.7)

28. Solve the rational inequality $\frac{x}{x+2} \geq 2$. (section 2.7)

29. Graph $f(x) = 2^{x-1} + 1$. Include the horizontal asymptote in your graph. (section 3.1)

30. Find the accumulated value of an investment of \$5,000 for 3 years at an interest rate of 6% if money is a) compounded quarterly and b) compounded continuously. (section 3.1)

31. Find the domain and vertical asymptote of $f(x) = \log_2(x-2)$ and then graph. Include the vertical asymptote in your graph. (section 3.2)

32. Use the properties of logarithms to expand the following logarithmic expression as much as possible. (section 3.3)
$\log_4\left(\frac{x^4\sqrt{x^2+3}}{64\,(x+3)^5}\right)$

33. Write the following logarithmic expression as a single logarithm whose coefficient is 1. (section 3.3)
$\frac{1}{2}\log_2 x + 2\log_2(y+2) - \frac{1}{3}\log_2 z$

34. Solve the exponential equation $9^{1-x} = \left(\frac{1}{27}\right)^{-2x+3}$. (section 3.4)

35. Solve the following equation. Round your solution to two decimal places. (section 3.4)
$e^{4x-5} - 7 = 11,243$

36. Solve the logarithmic equation $\log_3(x-5) + \log_3(x+3) = 2$. (section 3.4)

37. Solve the logarithmic equation $\log(x-3) + \log(x+1) = \log(7x-23)$. (section 3.4)

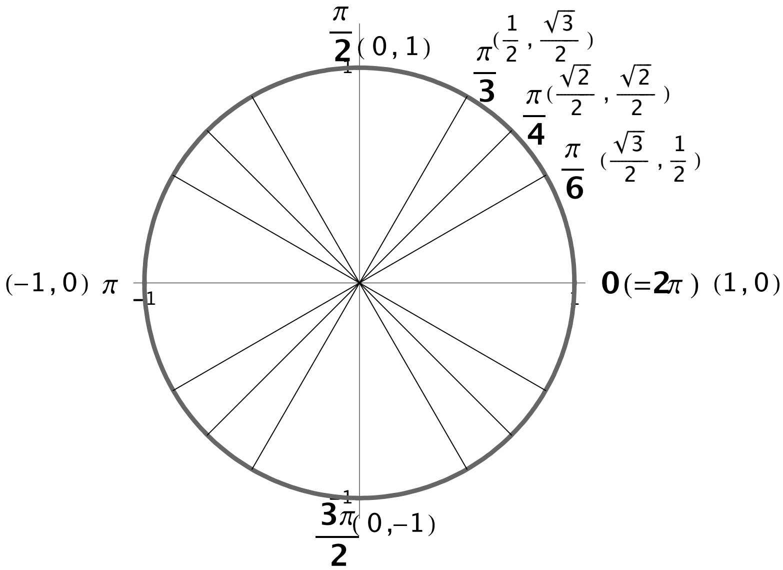
38. In 2003, the population in Mexico was approximately 104.9 million and by 2009 the population was approximately 116.2 million. a) Use the exponential growth model $A = A_0\,e^{kt}$, in which t is the number of years after 2003, to find an exponential growth function that models the data. Round k to three decimal places. b) What will the population be in 2015 to the nearest tenth of a million? c) When will population be 200 million to the nearest year? (section 3.5)

39. Use the exponential decay model $A = A_0\,e^{kt}$ to answer the following question. The half-life of thorium-229 is 7340 years. How long will it take for a sample of this substance to decay to 20% of the original amount? Round k to 6 decimal places and the number of years to one decimal place. (section 3.5)

40. The minute hand of a clock is 6 inches long and moves from 12 to 4 o'clock. How far does the tip of the minute hand move? Round your answer to two decimal places. (section 4.1)

41. A Ferris wheel of radius 25 feet makes 9 revolutions in 3 minutes. a) Find the linear speed, v, of a seat on this Ferris wheel to the nearest foot per minute. b) Find the angular speed, ω, of a seat on this Ferris wheel to the nearest radian per minute. (section 4.1)

42. Using the given reference angles, find the remaining angles and their coordinates on the unit circle. (section 4.2)



43. At a certain time of day, the angle of elevation of the sun is 40°. To the nearest foot, find the height of a tree whose shadow is 35 feet long. (section 4.3)

44. The point $(2, -3)$ is on the terminal side of an angle θ. Find the exact value of each of the six trigonometric functions. (section 4.4)

45. Find the exact value of each of the remaining trigonometric function of θ where $\cos\theta = -\frac{1}{3}$, θ in quadrant III. (section 4.4)

46. Find the exact values of the following trigonometric functions. Do not use a calculator.
a) $\sin\left(\frac{17\pi}{6}\right)$. b) $\tan\left(-\frac{9\pi}{4}\right)$. (section 4.4)

47. Determine the amplitude, period, and phase shift of $f(x) = 3\sin\left(2x - \frac{\pi}{2}\right)$ and then graph one period. (section 4.5)

48. Graph $y = \tan\left(x - \frac{\pi}{2}\right)$, $0 \le x \le 2\pi$. (section 4.6)

49. Find the exact value of a) $\sin^{-1}\left(-\frac{1}{2}\right)$ b) $\tan^{-1}\left(\sqrt{3}\right)$ c) $\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$ d) $\sec\left(\sin^{-1}\frac{1}{4}\right)$. (section 4.7)

50. Use a right triangle to write the following expression as an algebraic expression. Assume that x is positive and that the given inverse trigonometric function is defined for the expression in x. (section 4.7)
$\tan\left(\cos^{-1} 2x\right)$

51. An object moves in simple harmonic motion according to the function $d = -6\cos\frac{\pi}{4}t$, where t is measured in seconds and d in inches. Find a) the maximum displacement b) the frequency c) the time required for one cycle. (section 4.8)

52. A building that is 120 feet high casts a shadow that is 35 feet long. Find the angle of elevation of the sun to the nearest degree. (section 4.8)

53. Verify the following trigonometric identities. a) $\tan x + \cot x = \sec x \csc x$
b) $\frac{\cos x}{1-\sin x} = \sec x + \tan x$. (section 5.1)

54. Find the exact value of $\cos 105\,°$. (section 5.2)

55. Given $\tan\alpha = -\frac{4}{3}$, α lies in quadrant II, and $\cos\beta = \frac{2}{3}$, β lies in quadrant I, find a) $\cos(\alpha+\beta)$
b) $\sin(\alpha+\beta)$ c) $\tan(\alpha+\beta)$. (section 5.2)

56. Given $\tan\alpha = \frac{8}{15}$, α lies in quadrant III find a) $\sin 2\alpha$ b) $\cos 2\alpha$ c) $\tan 2\alpha$ d) $\sin\frac{\alpha}{2}$ e) $\cos\frac{\alpha}{2}$
f) $\tan\frac{\alpha}{2}$. (section 5.3)

57. Rewrite the expression $10\cos^4 x$ as an equivalent expression that does not contain powers of trigonometric functions greater than 1. (section 5.3)

58. Find a) all solutions in the interval $[0, 2\pi)$ and b) all solutions to the trigonometric equation
$2\cos^2 x + 3\cos x = -1$. (section 5.5)

59. To find the height of a building a surveyor takes a measurement from the west of a building and finds the angle of elevation to the top of the building is 62°. The surveyor takes another measurement from a point 200 feet to the west of the first point and finds the angle of elevation to the top of the building is 48°. Find the height of the building to the nearest foot. (section 6.1)

60. Points B and C are on opposite sides of a lake. From a point A on land, the distance to the point B is 1.25 miles and the distance to point C is 1.15 miles. If angle BAC is $55\,°$, find the length of the lake to the nearest hundreth of a mile. (section 6.2)

61. Solve the following system of equations. (section 7.1)

$$2x - 3y = 4$$
$$3x + 2y = 3$$

62. Solve the following system of equations. (section 7.4)

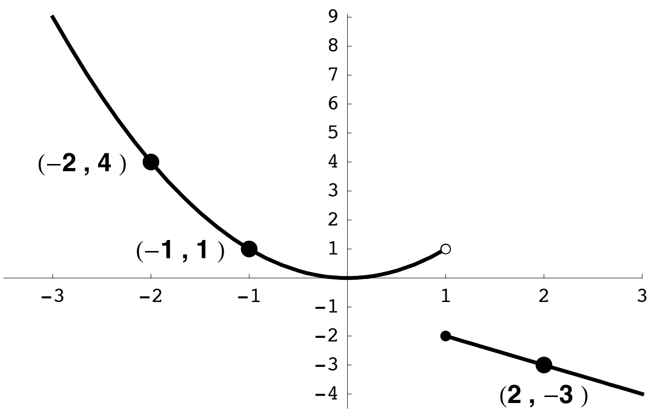
$$x^2 + y^2 = 5$$
$$3x - y = 5$$

63. Solve the following system of equations. (section 7.4)

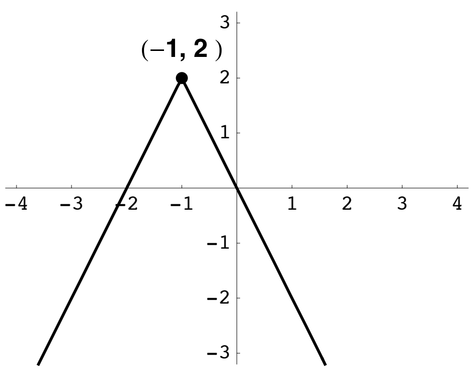
$$3x^2 + 2y^2 = 35$$
$$4x^2 + 3y^2 = 48$$

Solutions

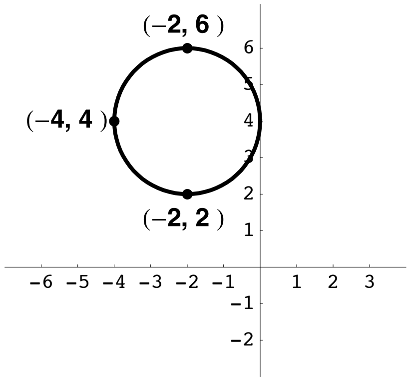
1. The relation is not a function. Domain: $\{2, 1, -2, 0\}$; Range: $\{4, 3, -2, 5\}$.
2. The equation does not determine y as a function of x. For instance, $(0, 5)$ and $(0, -5)$ both satisfy the equation.
3.



4. a) $-2x - h - 2$. b) $-\frac{1}{5x(x+h)}$.
5. $f(x)$ is neither even or odd by showing that $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$.
6. a) $(-\infty, \infty)$. b) $[-1, \infty)$. c) $-4, -2, 2, 4$. d) 2. e) increasing: $(-3, -1) \bigcup (3, \infty)$. decreasing: $(-\infty, -3) \bigcup (-1, 3)$. f) -1. The corresponding relative maximum is 3. g) -3 and 3. The corresponding relative minima are -1 and -1.
7. a) $y = \frac{2}{3}x + \frac{13}{3}$. b) $y = -\frac{3}{2}x$.
8. $\frac{f(3)-f(-2)}{3-(-2)} = -1$.
9.



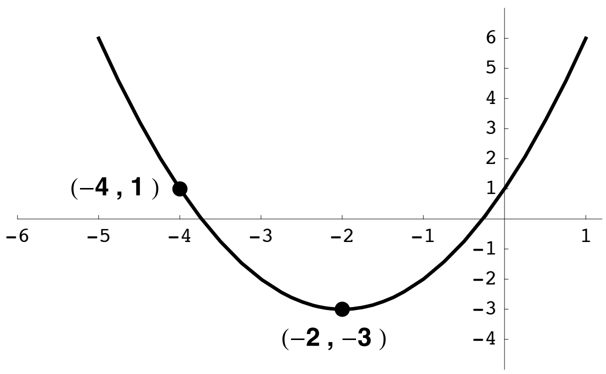
10. a) $(-\infty, -2) \bigcup (-2, 4) \bigcup (4, \infty)$. b) $(-\infty, 2]$.
11. $(f \circ g)(x) = \frac{6}{6+5x}$. Domain: $\left(-\infty, -\frac{6}{5}\right) \bigcup \left(-\frac{6}{5}, 0\right) \bigcup (0, \infty)$.
12. $f^{-1}(x) = \frac{x}{3x-2}$.
13. midpoint: $\left(-5, \frac{5}{2}\right)$. distance: $\sqrt{85}$.
14. $(x+2)^2 + (y-4)^2 = 4$. center $(-2, 4)$, radius 2.



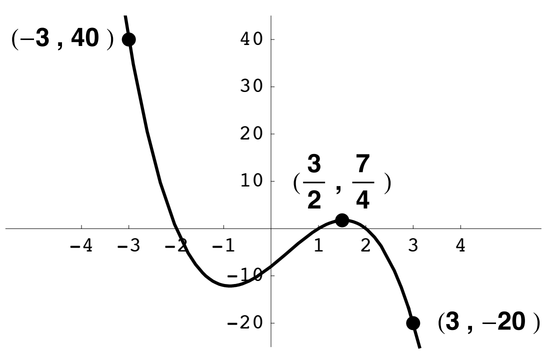
15. $-\frac{4}{25} - \frac{22}{25}\,i$.

16. $\left\{\frac{2}{3} + \frac{\sqrt{14}}{3}\,i,\ \frac{2}{3} - \frac{\sqrt{14}}{3}\,i\right\}$.

17.



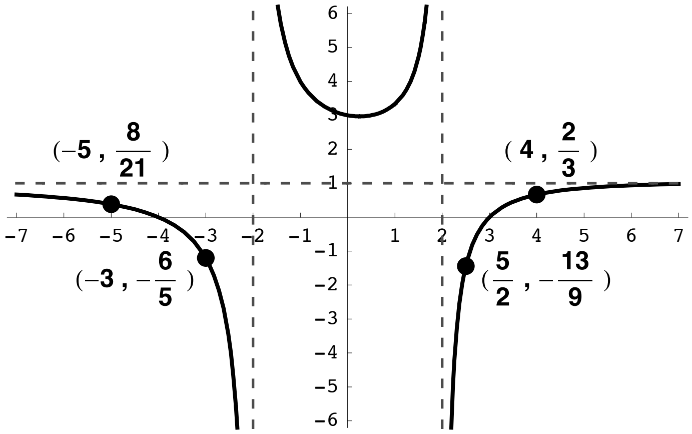
18. 50 feet × 25 feet. 1250 ft^2.
19. Up on the left and down on the right.
20. Since $f(x)$ is a polynomial function, $f(-3) = -11 < 0$, and $f(-2) = 1 > 0$, then by the Intermediate Value Theorem $f(x)$ has a real zero between -3 and -2.
21. $x-$intercepts: $-2, 1, 2$; $y-$intercept: -8.



22. $\pm\frac{8}{3}, \pm 8, \pm\frac{4}{3}, \pm 4, \pm\frac{2}{3}, \pm 2, \pm\frac{1}{3}, \pm 1$.

23. $\left\{3, \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

24. $f(x) = 3x^3 + 12x^2 - 93x - 522$.
25. $x-$intercept: -4, 3; $y-$intercept: 3; vertical asymptotes: $x = -2$, $x = 2$; horizontal asymptote: $y = 1$.

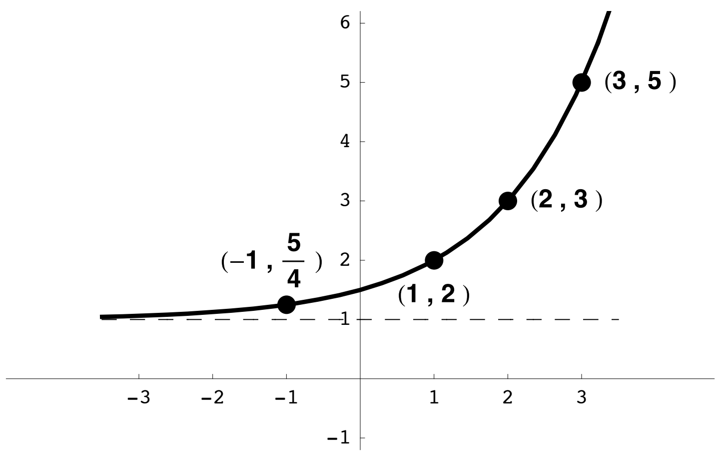


26. $y = 3x - 2$.

27. $(-\infty, -3] \cup \left[\frac{2}{3}, \infty\right)$.

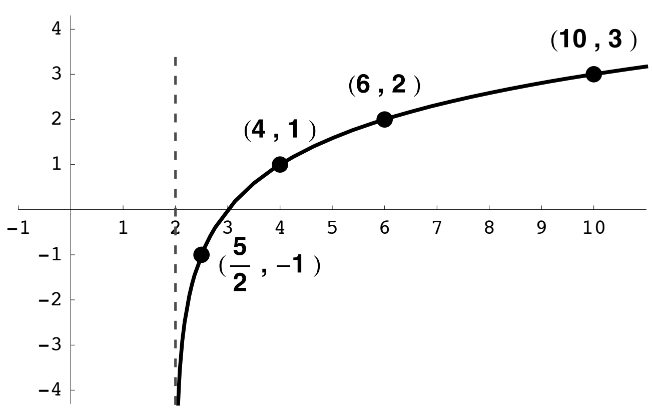
28. $[-4, -2)$.

29.



30. a) \$5978.09. b) \$5986.09.

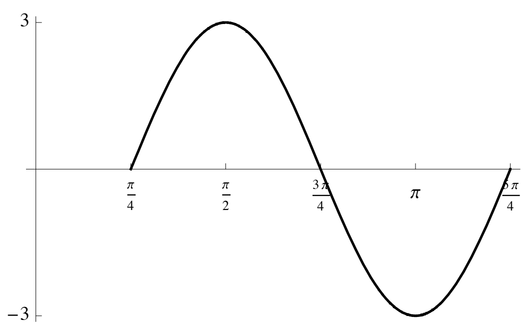
31. Domain: $(2, \infty)$. Vertical asymptote: $x = 2$.



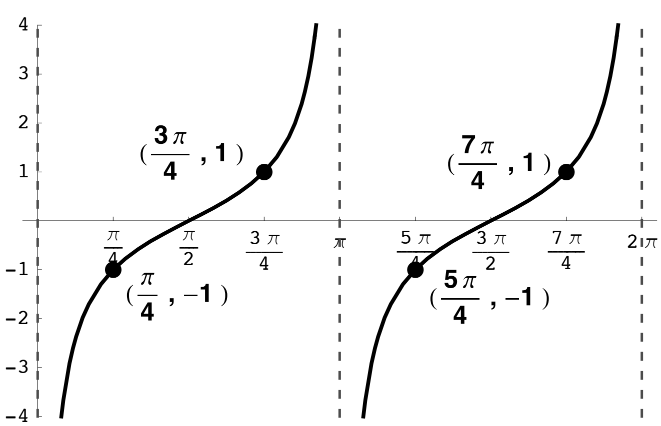
32. $4\log_4 x + \frac{1}{2}\log_4\left(x^2+3\right) - 3 - 5\log_4(x+3)$.

33. $\log_2 \frac{\sqrt{x}\,(y+2)^2}{\sqrt[3]{z}}$.

34. $x = \frac{11}{8}$.
35. 3.58.
36. $x = 6$.
37. $x = 4, 5$.
38. a) $A = 104.9\, e^{0.017\, t}$. b) 128.6 million. c) year 2041.
39. $A = A_0\, e^{-0.000094\, t}$; 17 121.7 years.
40. 12.57 inches.
41. a) $v = 471$ feet per minute. b) $\omega = 19$ radians per minute.
42. see textbook.
43. 29 feet.
44. $\sin\theta = -\frac{3\sqrt{13}}{13}$, $\cos\theta = \frac{2\sqrt{13}}{13}$, $\tan\theta = -\frac{3}{2}$, $\csc\theta = -\frac{\sqrt{13}}{3}$, $\sec\theta = \frac{\sqrt{13}}{2}$, $\cot\theta = -\frac{2}{3}$.
45. $\sin\theta = -\frac{2\sqrt{2}}{3}$, $\tan\theta = 2\sqrt{2}$, $\sec\theta = -3$, $\csc\theta = -\frac{3\sqrt{2}}{4}$, $\cot\theta = \frac{\sqrt{2}}{4}$.
46. a) $\frac{1}{2}$. b) -1.
47. Amplitude: 3; Period: π; Phase Shift: $\frac{\pi}{4}$.



48.



49. a) $-\frac{\pi}{6}$. b) $\frac{\pi}{3}$. c) $\frac{2\pi}{3}$. d) $\frac{4\sqrt{15}}{15}$.
50. $\frac{\sqrt{1-4x^2}}{2x}$.
51. a) 6 inches. b) $\frac{1}{8}$ cycles per second. c) 8 seconds.
52. 74°.
53. answer omitted.

54. $\frac{\sqrt{2}-\sqrt{6}}{4}$.

55. a) $\frac{-6-4\sqrt{5}}{15}$. b) $\frac{8-3\sqrt{5}}{15}$. c) $\frac{54-25\sqrt{5}}{22}$.

56. a) $\frac{240}{289}$. b) $\frac{161}{289}$. c) $\frac{240}{161}$. d) $\frac{4\sqrt{17}}{17}$. e) $-\frac{\sqrt{17}}{17}$. f) -4.

57. $\frac{15}{4} + 5\cos 2x + \frac{5}{4}\cos 4x$.

58. a) $x = \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$. b) $x = \frac{2\pi}{3} + 2n\pi,\ \pi + 2n\pi,\ \frac{4\pi}{3} + 2n\pi$, n any integer.

59. 542 feet.

60. 1.11 miles.

61. $\left(\frac{17}{13}, -\frac{6}{13}\right)$.

62. $\{(2, 1), (1, -2)\}$.

63. $\{(3, 2), (3, -2), (-3, 2), (-3, -2)\}$.

Trigonometric Identities

Sum Identities

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

Difference Identities

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

Double-Angle Identities

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Half-Angle Identities

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$

Power-Reducing Identities

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$