

1. Write the following complex numbers in standard form.

(a) $(-4 - 8i)(3 + i)$ Section 2.1, exercise 12:

$$\begin{aligned} -4 \cdot 3 + (-8i) \cdot 3 + (-4) \cdot i + (-8i)i &= -12 - 24i - 4i - 8i^2 \\ &= -12 - 28i + 8 = -4 - 28i \end{aligned}$$

(b) $\frac{-6i}{3 + 2i}$ Section 2.1, exercise 26:

$$\frac{-6i}{3 + 2i} \cdot \frac{3 - 2i}{3 - 2i} = \frac{-18i + 12i^2}{9 + 4} = \frac{-12 - 18i}{13} = \frac{-12}{13} - \frac{18}{13}i$$

(c) $\frac{3 - 4i}{4 + 3i}$ Section 2.1, exercise 28:

$$\frac{3 - 4i}{4 + 3i} \cdot \frac{4 - 3i}{4 - 3i} = \frac{12 - 9i - 16i + 12i^2}{16 + 9} = \frac{-25i}{25} = i$$

2. Find all solutions to the following equations.

(a) $x^2 - 6x + 10 = 0$ Section 2.1, exercise 45:

Apply the quadratic formula:

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$

This reduces to:

$$x = 3 \pm i$$

(b) $3x^2 = 8x - 7$ Section 2.1, exercise 49:

Rewrite the function:

$$3x^2 - 8x + 7 = 0$$

Apply the quadratic formula:

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(7)}}{2(3)} = \frac{4 \pm i\sqrt{5}}{3}$$

(c) $3x^2 = 4x - 6$ Section 2.1, exercise 50:

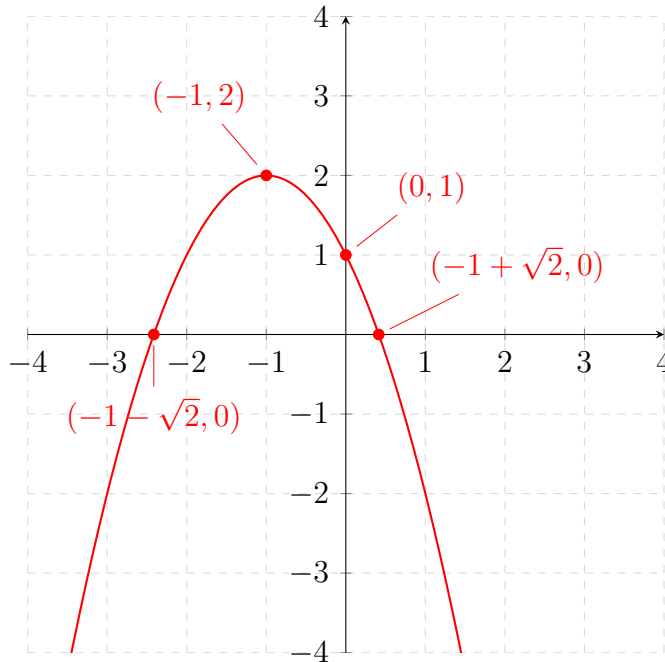
Rewrite the function:

$$3x^2 - 4x + 6 = 0$$

Apply the quadratic formula:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(6)}}{2(3)} = \frac{2 \pm i\sqrt{14}}{3}$$

3. Graph the following function: $f(x) = -x^2 - 2x + 1$. Mark on the graph (at the very least) the vertex, the x-intercepts (if any), and the y-intercept. p.305, example:



4. Find all zeroes of the following polynomials:

(a) $f(x) = x^3 + 3x^2 - x - 3$ p.322, example:

Factor by grouping:

$$f(x) = x^3 + 3x^2 - x - 3 = x^2(x+3) - (x+3) = (x^2-1)(x+3) = (x-1)(x+1)(x+3)$$

Zeroes: $x = -3, -1, 1$

(b) $f(x) = x^3 + x^2 - 5x - 2$ p.350, example:

Rational Zeroes Theorem: all possible rational zeroes: $x = \frac{\pm 1}{\pm 1}, \frac{\pm 2}{\pm 1} = 1, -1, 2, -2$.

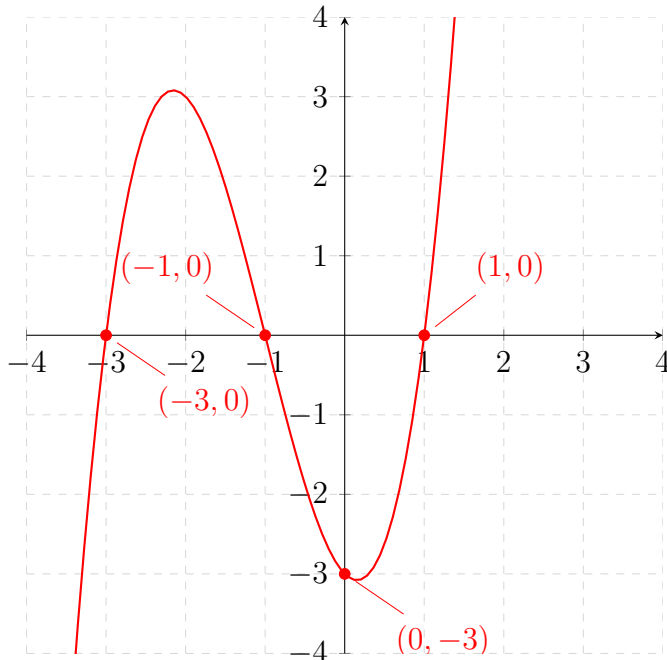
Test $x = 2$: $f(2) = 2^3 + 2^2 - 5(2) - 2 = 0$; so $(x - 2)$ is a factor of $f(x)$. Thus,

$$\begin{array}{r} x^2 + 3x + 1 \\ x - 2 \overline{) x^3 + x^2 - 5x - 2} \\ \underline{-x^3 + 2x^2} \\ 3x^2 - 5x \\ \underline{-3x^2 + 6x} \\ x - 2 \\ \underline{-x + 2} \\ 0 \end{array}$$

So $x^3 + x^2 - 5x - 2 = (x - 2)(x^2 + 3x + 1)$. The quadratic term has zeroes $x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{5}}{2}$; thus all possible zeroes are $x = 2, \frac{-3 \pm \sqrt{5}}{2}$

5. Graph the polynomial $f(x) = x^3 + 3x^2 - x - 3$.

It is already known from 4(a) where the zeroes are. The end behavior is given by the leading coefficient: rising to the right, falling to the left. Hence:



6. Find the n -th degree polynomial satisfying the given conditions:

(a) $n = 4$, -2 , $-\frac{1}{2}$, i are zeroes; $f(1) = 18$ p.357, exercise 30:

i is a zero means $-i$ is a zero.

So $f(x) = a(x - (-2))(x - (-\frac{1}{2}))(x - i)(x - (-i))$.

Multiplying terms with imaginary elements we have

$$f(x) = a(x + 2)(x + \frac{1}{2})(x^2 + 1)$$

We are told $18 = f(1) = a(1 + 2)(1 + \frac{1}{2})(1^2 + 1) = a(3)(\frac{3}{2})(2)$; hence $a = 2$ and

so $f(x) = 2(x + 2)(x + \frac{1}{2})(x^2 + 1)$; alternatively, $f(x) = (x + 2)(2x + 1)(x^2 + 1)$.

(b) $n = 4$, -2 , 5 , and $3 + 2i$ are zeroes; $f(1) = -96$ p.357, exercise 31

$3 + 2i$ is a zero means $3 - 2i$ is a zero.

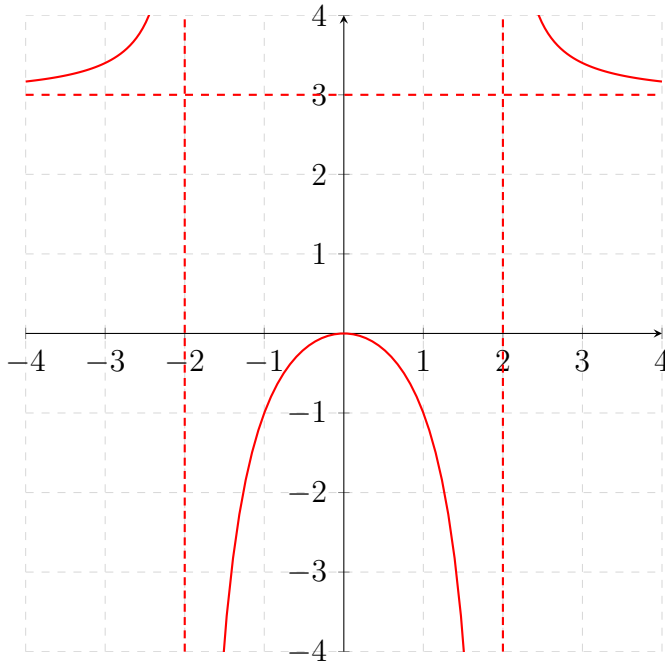
So $f(x) = a(x - (-2))(x - 5)(x - (3 + 2i))(x - (3 - 2i))$.

Multiplying terms with imaginary elements we have

$$f(x) = a(x + 2)(x - 5)(x^2 - 6x + 13)$$

We are told $-96 = f(1) = a(1 + 2)(1 - 5)(1^2 - 6(1) + 13) = a(3)(-4)(8) = a(-96)$; hence $a = 1$ and so $f(x) = (x + 2)(x - 5)(x^2 - 6x + 13)$

7. Graph $f(x) = \frac{3x^2}{x^2 - 4}$. p.370, example 6:



8. Find the slant asymptote of $f(x) = \frac{x^2 - 4x - 5}{x - 3}$. p.373, example 8:

$$\begin{array}{r}
 x - 1 \\
 x - 3 \overline{) x^2 - 4x - 5} \\
 \underline{-x^2 + 3x} \\
 -x - 5 \\
 \underline{x - 3} \\
 -8
 \end{array}$$

The slant asymptote is given by $y = x - 1$.

9. Solve the inequality $x^3 + x^2 \leq 4x + 4$. p.385, example 3:

Rewrite so that 0 is on one side: $x^3 + x^2 - 4x - 4 \leq 0$.

Factor by grouping: $x^2(x + 1) - 4(x + 1) = (x + 1)(x^2 - 4) = (x + 1)(x - 2)(x + 2) \leq 0$

Test by positives and negatives: from $-\infty$ to -2 , the function is negative. From -2 to -1 the function is positive. From -1 to 2 the function is negative, and from 2 to ∞ the function is positive. Hence our solution set is $(-\infty, -2] \cup [-1, 2]$

10. Solve the inequality $\frac{x - 2}{x + 2} \geq 2$. p.391, exercise 59 (modified):

Rewrite: $\frac{x - 2}{x + 2} - 2 \geq 0$; $\frac{x - 2 - 2(x + 2)}{x + 2} = \frac{-x - 6}{x + 2} \geq 0$. Test by positives and negatives: from $-\infty$ to -6 the function is negative. From -6 to -2 the function is positive. From -2 to ∞ the function is negative. However, note that -2 is not a valid solution because of division by zero; hence, our answer is $[-6, -2)$.