1. True or false: if false, provide a counterexample.

   (a) True/false: In general, my address book defines a *function* from phone numbers to people.
   This one is ambiguous, and was thrown out from grading: a family with one house phone is an example where one phone number may lead to multiple people

   (b) True/false: The graph of (time of day, calories I've burned) is a horizontal line.
   p. 152, #70: No. Even while sleeping the human body burns calories.

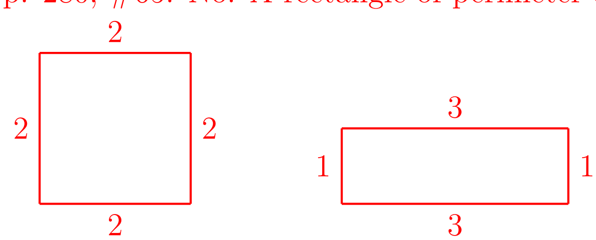
   (c) True/false: If $f(x) = c$, where c is a constant, the difference quotient of f is always one.
   No: $\dfrac{f(x+h)-f(x)}{h} = \dfrac{c-c}{h} = \dfrac{0}{h} = 0$

   (d) True/false: In general, if $Ax + By + C = 0$, the slope is A.
   No: $By = -Ax - C \implies y = \dfrac{-A}{B} - \dfrac{C}{B}$

   (e) True/false: In general, $f + g$ is the same as $g + f$.
   Yes: addition is commutative

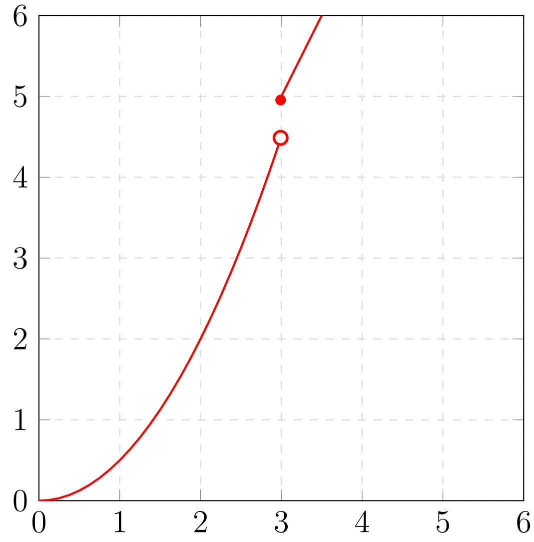
   (f) True/false: In general, $f \circ g$ is the same as $g \circ f$.
   No: as a counterexample, consider $f(x) = \dfrac{1}{x}$ and $g(x) = x + 1$: $f \circ g = \dfrac{1}{x+1}$ and $g \circ f = \dfrac{1}{x} + 1$.

   (g) True/false: Knowing the perimeter of a rectangle is the same as knowing the area.
   p. 280, #65: No. A rectangle of perimeter 8 could, for example, be either



2. Graph the following: p.184, #51

$$f(x) = \begin{cases} \frac{1}{2}x^2 & \text{if } x < 3 \\ 2x - 1 & \text{if } x \geq 3 \end{cases}$$



3. Write the equation for the line that passes through $(1, 1)$ that is perpendicular to the line $y = 2x - 1$.
   Perpendicular line must have slope $\dfrac{-1}{2}$. Using point-slope form, this means the equation is $(y - 1) = \dfrac{-1}{2}(x - 1)$. Further algebra yields $y = \dfrac{-x}{2} + \dfrac{3}{2}$

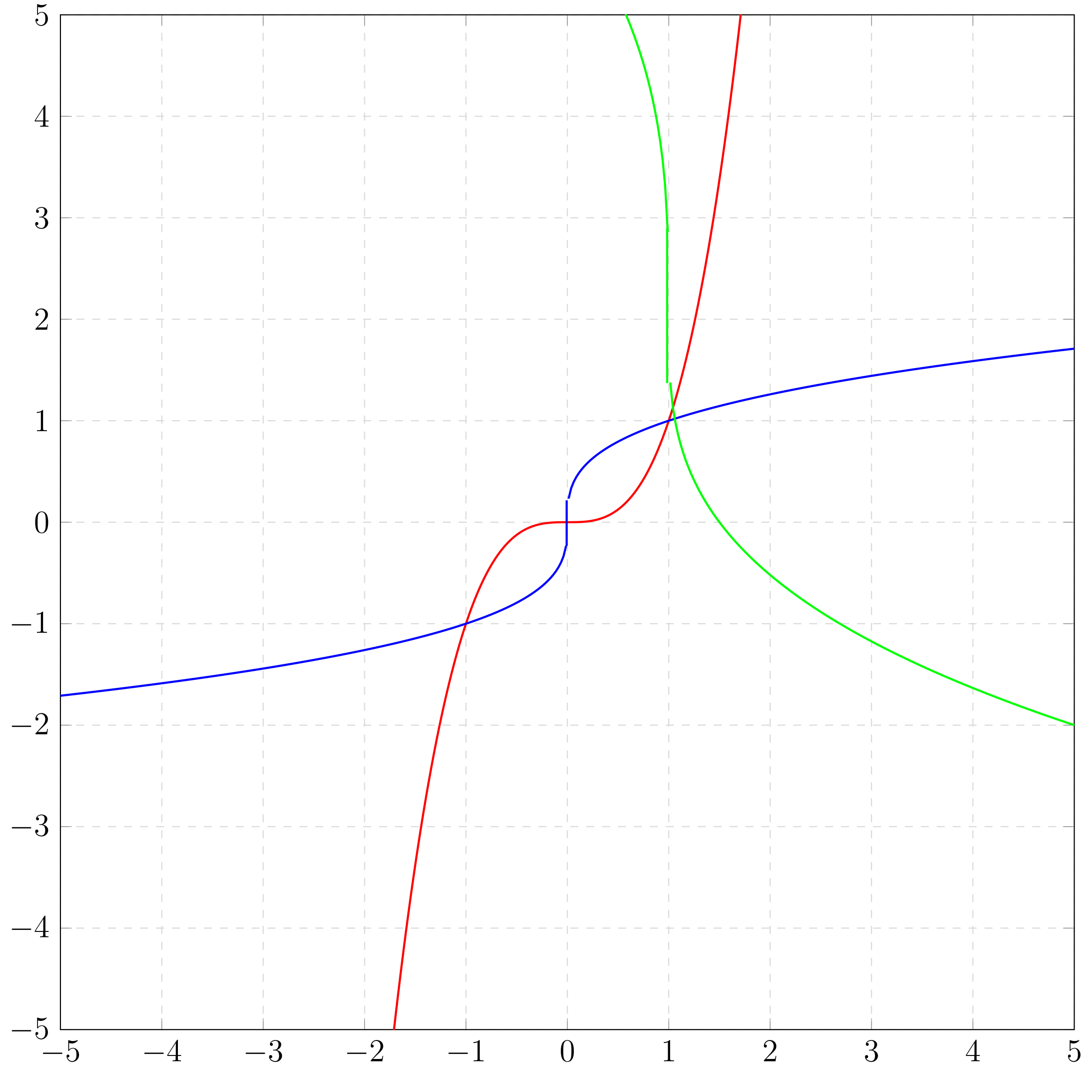
4. Write the equation for the line that passes through $(3, 0)$ that is perpendicular to your answer from problem 3.
   Perpendicular line to the answer from 3 means we have slope 2. Using point-slope form, this means the equation is $(y - 0) = 2(x - 3)$. Further algebra gives us $y = 2x - 6$.

5. Graph the following on the same graph:

   (a) The function $f(x) = x^3$. x^3

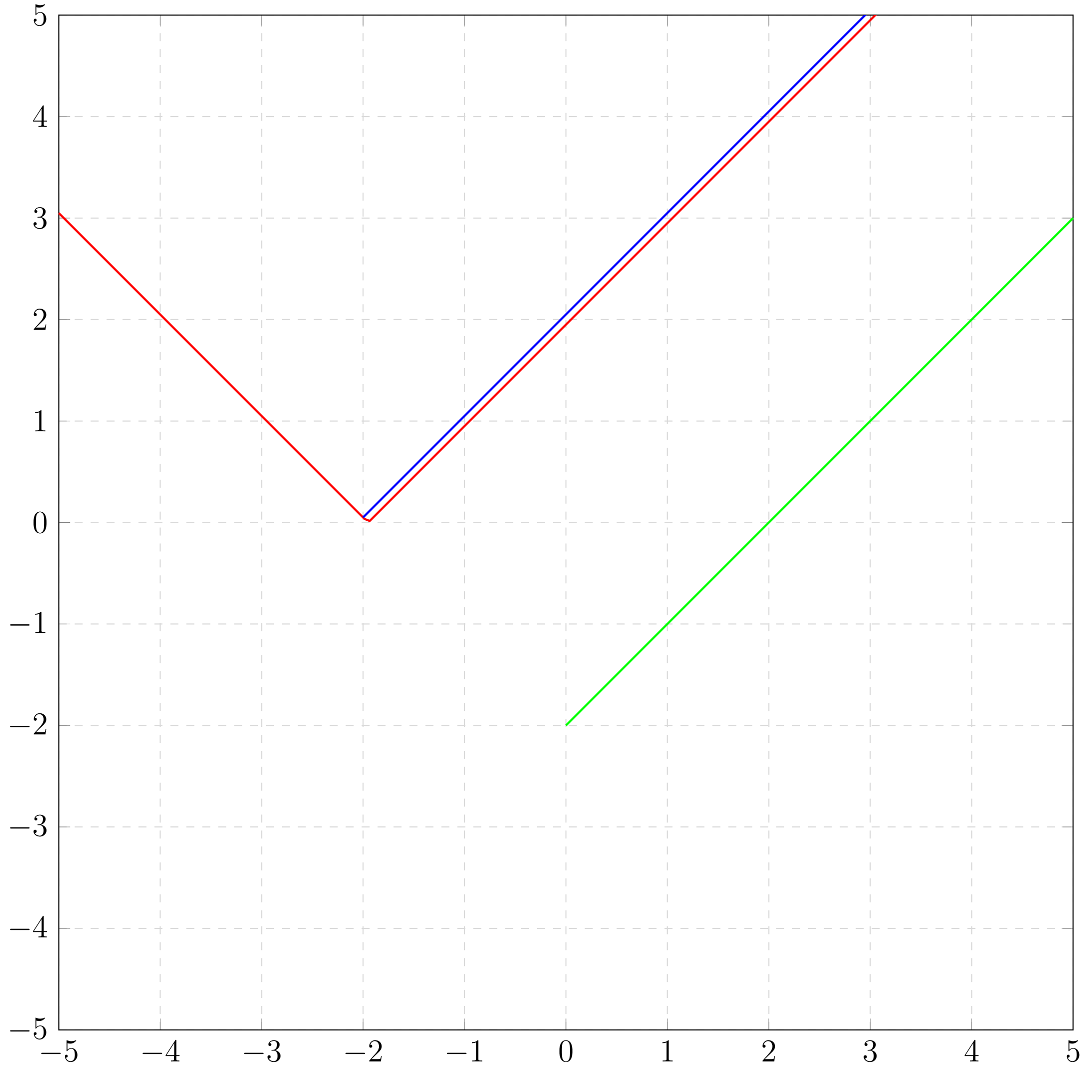
   (b) The function $f^{-1}(x) = \sqrt[3]{x}$. $\sqrt[3]{x}$

   (c) The transformed function $g(x) = -2f^{-1}(2(x-1)) + 2 = -2\sqrt[3]{2(x-1)} + 2$ $-2\sqrt[3]{2(x-1)} + 2$



6. Determine the inverse of $f(x) = \frac{5}{x} + 4$. p.250, example 4: $y = \frac{5}{x} + 4 \implies x = \frac{5}{y} + 4 \implies x - 4 = \frac{5}{y} \implies \frac{1}{x-4} = \frac{y}{5} \implies \frac{5}{x-4} = y \implies f^{-1}(x) = \frac{5}{x-4}$

7. Given $f(x) = |x+2|$, determine and graph an inverse function for f. First, the function does not pass the horizontal line test. Thus it is necessary to modify the domain such that $f(x)$ is one-to-one. The most convenient domain is $[0, \infty)$. In blue, the restricted function, and in green, the inverse function.



8. Given $f(x) = \sqrt{x-1}$, $g(x) = x^2$:

   (a) $(f \circ g)(x) = \sqrt{(x^2) - 1}$

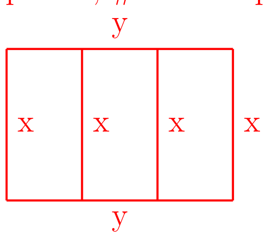
   (b) Domain: The domain has to be the domain of g restricted such that $g(x)$ is in $f(x)$'s domain. f has domain $x \geq 1$, so we are looking for all x such that $g(x) \geq 1$. This means we are looking for x such that $x^2 \geq 1$; this is $\{x \geq 1\} \bigcup \{x \leq -1\}$.

   (c) $(g \circ f)(x) = \left(\sqrt{x-1}\right)^2 = x - 1$

   (d) Domain: The domain here is the domain of f restricted such that $f(x)$ is in g's domain. Since g has all real numbers as domain, this is just the domain of f: $\{x \geq 1\}$.

9. Find the center and radius of the circle given by $x^2 + y^2 + 6x + 2y + 6 = 0$.
   p. 264, #53: Complete the square. $x^2+y^2+6x+2y+6 = (x+3)^2-3^2+(y+1)^2-1^2+6 = 0 \implies (x+3)^2+(y+1)^2-10+6 = 0 \implies (x+3)^2+(y+1)^2 = 4$. This implies that the center and radius of the circle are (-3,-1), radius 2.

10. You have 1200 feet of fencing to enclose a rectangular region and subdivide it into three smaller rectangular regions by placing two fences parallel to one of the sides. Express the area of the enclosed region, A, as a function of one of its dimensions, x.
    p. 278, #26: The problem boils down to drawing this diagram:



    The equations: $1200 = 4x + 2y$ and $A = xy$. $1200 = 4x + 2y \implies 2y = 1200 - 4x \implies y = 600 - 2x$, so $A = x(600 - 2x)$

11. You invested \$8000, part of it in a stock that paid 12% annual interest. However, the rest of the money suffered a 5% loss. Express the total annual income from both investments, I, as a function of the amount invested in the 12% stock, x.
    Of the original \$8000, x of it goes to the 12% account, and $8000-x$ goes to the account that loses 5%. Thus, our *income* was $I(x) = .12x - .05(8000 - x)$

12. Consider the three points $A : (1, 1 + d)$, $B : (3, 3 + d)$, $C : (6, 6 + d)$. p.266, #94

    (a) Express the distance from A to B. $\sqrt{(1-3)^2 + (1 + d - (3 + d))^2} = \sqrt{4+4} = \sqrt{8}$

    (b) Express the distance from B to C. $\sqrt{(3-6)^2 + (3 + d - (6 + d))^2} = \sqrt{9+9} = \sqrt{18}$

    (c) Express the distance from A to C. $\sqrt{(1-6)^2 + (1 + d - (6 + d))^2} = \sqrt{25+25} = \sqrt{50}$

    (d) Are the points A, B, and C collinear (that is, on the same line)? Why or why not? (*Hint: think about how the sides of a triangle relate to its hypoteneuse.*) Yes. Any path from A to C is at the very least the same length as the distance from A to C; the path from A to C through point B is in fact the same length as the distance from A to C; hence B must be on the straight line from A to C.

13. Given the function $f(x) = \frac{2}{x}$:

    (a) What is the average rate of change for this function from $x_0 = 1$ to $x_0 = 2$?
    $$\frac{f(2) - f(1)}{2 - 1} = \frac{\frac{2}{2} - \frac{2}{1}}{2 - 1} = \frac{1 - 2}{2 - 1} = -1$$

    (b) What is the difference quotient for this function? $\frac{f(x+h) - f(x)}{h} = \frac{\frac{2}{x+h} - \frac{2}{x}}{h} =$
    $$\frac{\frac{2x}{x(x+h)} - \frac{2(x+h)}{x(x+h)}}{h} = \frac{\frac{-2h}{x(x+h)}}{h} = \frac{-2}{x(x+h)}$$